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Name of College - S.S. College.

J-Bad

Deptt - Mathematics

Topic - Ellipse (Contd) B.Sc I (Sub)

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Time - 1 - 1.45

* Condition under which the St. line $y = mx + c$ will Touch the Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Solution → Let The St. line $y = mx + c$ (I)

Meet the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{--- (II)}$$

in two points P and Q.

The Co-ordinate of the point where the St. line (I) meets (II) must satisfy both Equations.

Hence for the common points, eleminate y from (I) and (II)

We have

$$\frac{x^2}{a^2} + \frac{(mx+c)^2}{b^2} = 1$$

$$b^2x^2 + a^2(m^2x^2 + 2mxc + c^2) - a^2b^2 = 0$$

$$\Rightarrow (a^2m^2 + b^2)x^2 + 2mxca^2 + a^2(c^2 - b^2) = 0 \quad \text{--- (3)}$$

The x-co-ordinates of the two points of intersection will be the roots of (3)

If the line will be tangent

then $\Delta = 0$

$$\Rightarrow (2a^2mc)^2 - 4(a^2m^2+b^2)(c^2-b^2)a^2 = 0$$

$$\Rightarrow 4a^4m^2c^2 - 4a^2(a^2m^2+b^2)(c^2-b^2) = 0$$

$$\Rightarrow \cancel{a^2m^2c^2} - \cancel{a^2m^2c^2} + a^2b^2m^2 - b^2c^2 + b^4 = 0$$

$$\Rightarrow a^2m^2 - c^2 + b^2 = 0$$

$$c^2 = a^2m^2 + b^2$$

$$c = \sqrt{a^2m^2+b^2}$$

Thus whatever m be

the line

$$y = mx + \sqrt{a^2m^2+b^2}$$

will touch the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

To find the co-ordinates of point of contact ——

Find the point of contact of the tangent $lx + my + n = 0$ to the ellipse.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Solution

Let the point of contact be (α, β)

Therefore the equation of tangent at (α, β) be

$$\frac{x\alpha}{a^2} + \frac{y\beta}{b^2} = 1 \quad \text{--- (1)}$$

Since $lx + my + n = 0$ --- (ii) is tangent to the ellipse.

$\therefore$ (1) and (ii) are identical

Hence $$\frac{\frac{\alpha}{a^2}}{l} = \frac{\beta}{b^2 m} = -\frac{1}{n}$$

$$\Rightarrow \frac{\alpha}{a^2 l} = \frac{\beta}{b^2 m} = -\frac{1}{n}$$

$$\left.\begin{aligned} \Rightarrow \alpha &= -\frac{a^2 l}{n} \\ \beta &= -\frac{b^2 m}{n} \end{aligned}\right\}$$ this gives the point of contact

Since (α, β) lies on ellipse.

Therefore $$\frac{\alpha^2}{a^2} + \frac{\beta^2}{b^2} = 1$$

$$\frac{a^4 l^2}{n^2 a^2} + \frac{b^4 m^2}{n^2 b^2} = 1$$

$$\frac{a^2 l^2}{n^2} + \frac{b^2 m^2}{n^2} = 1$$

$$\Rightarrow a^2 l^2 + b^2 m^2 = n^2$$

Which is the required condition under which $lx + my + n = 0$ will be tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Find the equation of normal at the point (α, β) of the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Since Tangent and Normal both are mutual perpendicular at the point of contact (α, β)

As slope of the tangent

$$=\left(\frac{dy}{dx}\right)_{\alpha,\beta}$$

$$=-\frac{b^2}{a^2}\frac{\alpha}{\beta}$$

$\therefore$ Slope of the Normal $=\frac{a^2\beta}{b^2\alpha}$

$\therefore$ Equation of Normal

$$\frac{y-\beta}{x-\alpha}=\frac{a^2\beta}{b^2\alpha}$$

$$\frac{y-\beta}{a^2\beta}=\frac{x-\alpha}{b^2\alpha}$$

$$\Rightarrow \frac{y-\beta}{\frac{\beta}{b^2}}=\frac{x-\alpha}{\frac{\alpha}{a^2}}$$